DEFINITIONS

Graph Terminology

- A graph *G* consists of two finite sets: a set *V(G)* of vertices and a set *E(G)* of edges, where each edge is associated with a set consisting of either one or two vertices called its endpoints.
- The correspondence from edges to the set of endpoints is called the edge-endpoint function.
- An edge with just one endpoint is called a loop.
- Two distinct edges with the same set of endpoints are said to be parallel.
- An edge is said to connect to its endpoint; two vertices that are connected by an edge are called adjacent; and a vertex that is an endpoint of a loop is said to be adjacent to itself.
- An edge is said to be incident on each of its endpoints, and two edges incident on the same endpoint are called adjacent.
- A vertex on which no edges are incident is called isolated.
- A graph with no vertices is called empty, and one with at least one vertex is called non-empty. I.e. *G* is empty iff $V(G)=\emptyset$

Directed Graphs

- A directed graph, or digraph *G*, consists of two finite sets: a set *V(G)* of vertices and a set *D(G)* of directed edges, where each is associated with an ordered pair of vertices called its endpoints.
- If edge *e* is associated with the pair (v,w) of vertices, then *e* is said to be the (directed) edge from *v* to *w*.

Subgraphs

- A graph *H* is said to be a subgraph of a graph *G* if and only if

  $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$

SIMPLE GRAPHS

Simple Graphs

- A simple graph is a graph that does not have any loops or parallel edges.
- In a simple undirected graph, an edge with endpoints v and w is denoted $\{v,w\}$.

Complete Graphs

- Let n be a positive integer. A complete graph on n vertices, denoted K_n, is a simple graph with n vertices $v_1, v_2, \ldots, v_n,$ whose set of edges contains exactly one edge for each pair of distinct vertices.

Graph complements

If G is a simple graph, the complement of G, denoted G' is the simple graph defined as follows:

- $V(G') = V(G)$
- $E(G) \cap E(G') = \emptyset$
- The graph whose vertex set is $V(G)$ and set of edges is $E(G) \cup E(G')$ is complete.

DEGREE OF GRAPHS

Definitions

Let G be a graph

- Let v be a vertex of G. The degree of v, denoted $\deg(v)$, equals the number of edges that are incident on v, with an edge that is a loop counted twice.
- The total degree of G is the sum of the degree of all the vertices of G

Handshake Theorem

- Handshake Theorem:
  For any graph G,
  the total degree of G equals twice the number of edges of G.
  I.e. if $V(G) = \{ v_1, v_2, \ldots, v_n\}$, where n is a non-negative integer, then
  Total degree of G $= \deg(v_1) + \deg(v_2) + \ldots + \deg(v_n)$
  $= 2$ (the number of edges of G)
- Corollary: The total degree of a graph is even
- Corollary: In any graph, the number of vertices of odd degree is even.